**Comparison of classical parametric and nonparametric inferential methods with the bootstrap resampling method**

**Name: Soumya Mukherjee**

**Sem: VI**

**Roll: 449**

**Under the guidance of Prof. Debjit Sengupta**

***I affirm that I have identified all my sources and that no part of my dissertation paper uses unacknowledged materials.***

**Signature**: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. **Introduction**

Statistical inference is concerned with inferring about the unknown characteristics of a population under study on the basis of limited information available from such a population in the form of a random sample drawn from it. Based on the sample data, certain descriptive measures or statistics are obtained .Using the knowledge of probability theory and the sampling distribution of the statistics used, valid statements regarding the unknown population characteristic may be made. Statistical inference is primarily of two paradigms – parametric inference and nonparametric inference.

In parametric inference, the population from which the sample is drawn is assumed to be well modelled by a probability distribution possibly belonging to a family of distributions , each indexed or labelled by a parameter , say . Here, the exact functional form of the underlying probability distribution is completely known except for the parameter .Our objective is to infer about the plausible values of the parameter in the light of the data at hand, known as the estimation problem, or to verify whether a hypothesised value or range of values of the parameter is plausible, known as the problem of testing of hypothesis. Estimation, again, may be of two types. We may estimate either by a single point value, known as point estimation, or obtain a range of plausible values of , based on the data at hand.

Nonparametric inference, on the other hand, is not based on the assumption that the underlying population is well modelled by a parametrized family of distributions. In many real life problems, the functional form of the distribution is hardly ever known. It is therefore desirable to develop methods that are based on quite general assumptions about the underlying distribution, which are known as non-parametric or distribution-free methods and inferential procedures based on such methods is known as nonparametric inference. An inferential procedure is distribution-free if the statistic used has a distribution which does not depend upon the distribution of the population from which the sample is drawn. Further, problems under the purview of nonparametric inference are not concerned with the values of any parameter as in the case of parametric inference.

Classical methods of estimation and hypothesis-testing are based on the properties of the sampling distribution of estimators. In many situations the sampling distribution of an estimator may be mathematically intractable or the idealized assumptions of classical statistics such as normality and large sample size may not hold. In such cases the method of bootstrapping may be employed.

Bootstrapping is a general simulation-based resampling method used to estimate the sampling distribution of estimators, as well provide an accurate idea of the standard error and bias of different statistics. It also provides reliable confidence intervals and enables us to perform statistical hypothesis testing. Bootstrapping can be employed in both parametric and nonparametric setup, respectively known as parametric bootstrap and nonparametric bootstrap.

The basic idea behind the bootstrap is to develop an approximation to the true sampling distribution of the statistic of interest, on the basis of a single data set. The bootstrap philosophy may be summarised as: “The population is to the sample as the sample is to the resample/bootstrap sample”. In case of nonparametric bootstrap, resamples are drawn by drawing random samples with replacement from the sample data at hand and the sampling distribution of the statistic is estimated by constructing the empirical distribution of the statistic based on the values of the statistic computed from each of the resamples. In case of parametric bootstrap, an estimate of the parametric model under study is first constructed (often but not invariably by maximum likelihood method) based on the sample data. Resamples are drawn by drawing random samples with replacement from this estimated parametric model and on the basis of the values of the statistic computed from each of the resamples, the sampling distribution of the statistic is estimated. Once the sampling distribution has been estimated, measures of accuracy of the estimate such as bias and standard error can be computed based on it. Further interval estimation and hypothesis testing may be performed.

Our objective in this paper is to discuss the application of the bootstrap technique to construct point and interval estimates of the location parameter of the underlying distribution by simulating observations from the Normal, Exponential, Cauchy and Poisson distributions under the non parametric setup. Under the parametric setup, point and interval estimates of the mean parameters of the Normal, Exponential and Poisson distributions, the median of the Cauchy distribution and the pre-truncation mean of the zero-truncated Poisson distribution are constructed. Wherever possible, results obtained using classical inferential techniques are compared with those obtained using the bootstrap technique. Point estimates are compared in terms of their standard errors and mean square errors (MSE’s) and confidence intervals are compared in terms of their average lengths and empirical coverages.

The paper is organised as follows. Section 1 contains the introduction..Section 2 contains the theory behind the bootstrap technique. Section 3 contains the theoretical procedures and results used to apply classical procedures. Section 4 contains details regarding the simulation studies. Section 5 contains tables and results. Section 6 contains the concluding discussions. Finally Section 7 contains the references.

1. **The theory behind bootstrap**

We first illustrate the general theory behind the non parametric bootstrap and then discuss how it can be modified to the parametric setup.

**Non-parametric bootstrap:**

Problems of statistical inference often involve estimating some aspect of a probability distribution of a random variable X on the basis of a random sample drawn from *.* If no assumptions are made regarding the functional form of as is the case in nonparametric inference,the empirical distribution function, which we will call *,* is a simple estimate of the entire distribution *.* An obvious way to estimate some interesting aspect of *,* like its mean or median or correlation, is to use the corresponding aspect of *.* This is the "plug-in principle." The bootstrap method is a direct application of the plug-in principle,

**The empirical distribution function**

Having observed a random sample of size *n* from a probability distribution *,*

the *empirical distribution function* is defined to be the discrete distribution that puts probability on each value

In other words, assigns to a set *A* in the sample space of *x* its empirical probability

the proportion of the observed sample occurring in *A.* The hat symbol always indicates quantities calculated from the observed data.

A parameter is a function of the probability distribution *.* A statistic is a function of the sample . We will sometimes write parameters directly as functions of *,* say

This notation emphasizes that the value of the parameter is obtained by applying some numerical evaluation procedure to the distribution function *.* For example if is a probability distribution in the real line, the expectation can be thought of as the

parameter

where X is a random variable having the distribution .

**The plug-in principle**

The plug-in principle is a simple method of estimating parameters from samples. The *plug-in estimate* of a parameter is defined to be . In other words, we estimate the function of the probability distribution *F* by the same function of the empirical distribution *,* .

In general, the plug-in estimate of an expectation is,

A question may arise as regards to the efficiency in using the plug-in principle. It is usually quite good, if the only available information about comes from the sample . Under this circumstance cannot be improved upon as an estimator *,* at least not in the usual asymptotic sense of statistical theory.

We will use the bootstrap to study the bias and standard error of the plug-in estimate

*.* The bootstrap's virtue is that it produces biases and standard errors in an automatic way, no matter how complicated the functional mapping may be. We will see that the bootstrap itself is an application of the plug-in principle.

The plug-in principle is less good in situations where there is information about *F* other than that provided by the sample x. We might know, or assume, that is a member of a parametric family*.* In that case we use the parametric bootstrap.

**The bootstrap estimate of standard error:**

Let a random sample be drawn from and be the realization of**.** We wish to estimate a parameter of interest on the basis of**.**

For this purpose, we calculate an estimate from (Note that may be the plug-in estimate *,* but it doesn't have to be). We are interested to know the accuracy of .The bootstrapwas introduced in 1979 as a computer-based method for estimating the standard error of*.* It enjoys the advantage of being completely automatic. The bootstrap estimate of standard error requires no theoretical calculations, and is available no matter how mathematically complicated the estimator may be.

Bootstrap methods depend on the notion of a bootstrap sample. Let be the empirical distribution, putting probability on each of the observed values

A bootstrap sample is defined to be a random sample of size *n* drawn from *,* say

, The star notation indicates that is not the actual data set , but rather a randomized, or resampled,version of . We may view the bootstrap data points , as a random sample of size *n* drawn with replacement from the population of *n* objects *·*

The bootstrap data set , consists of members of the original data set *·,* some appearing zero times, some appearing once, some appearing twice, etc.

Corresponding to a bootstrap data set x\* is a bootstrap replication of *,*

The quantity is the result of applying the same function to as was applied to . For example if is the sample mean then is the mean of the bootstrap data set,

The bootstrap estimate of, the standard error of a statistic *,* is a plug-in estimate that uses the empirical distribution function in place of the unknown distribution *.* Specifically, the bootstrap estimate of is defined by . In other words, the bootstrap estimate of is the standard error of for data sets of size *n* randomly sampled from *.*

is called the ideal bootstrap estimate of standard errorof *.* Unfortunately, for virtually any estimate other than the mean, there is no neat formula that enables us to compute the numerical value of the ideal estimate exactly.

The bootstrap algorithm, described next, is a computational way of obtaining a good approximation to the numerical value of .

It is easy to implement bootstrap sampling using a computer. A random number device selects integers each of which equals any value between 1 and *n* with probability . The bootstrap sample consists of the corresponding members of ,

The bootstrap algorithm works by drawing many independent bootstrap samples, evaluating the corresponding bootstrap replications, and estimating the standard error of by the empirical standard deviation of the replications. The result is called the bootstrap

estimate of standard error, denoted by where *B* is the number of bootstrap samples used.

Algorithm 6.1 is a more explicit description of the bootstrap procedure for estimating the standard error of from the observed data ,

**The bootstrap algorithm for estimating standard errors**

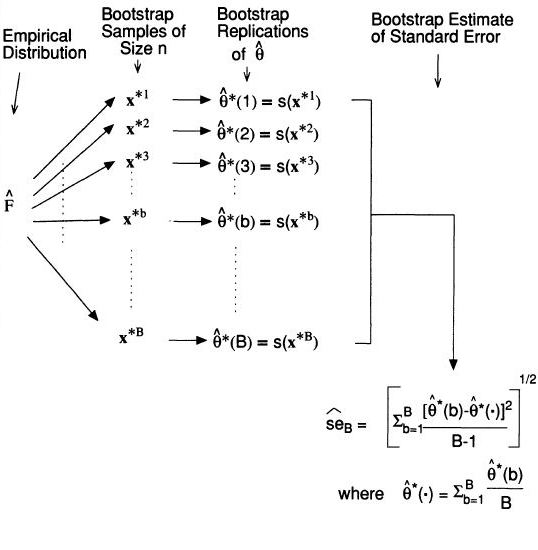
1. Select *B* independent bootstrap samples *,* each consisting of *n* data values drawn with replacement from using a random number device(According to Efron , for estimating a standard error, the number *B* will ordinarily be in the range 25- 200)
2. Evaluate the bootstrap replication corresponding to each bootstrap sample,
3. Estimate the standard error by the sample standard deviation of the *B* replications

where

The limit of as *B* goes to infinity is the ideal bootstrap estimate of ,

The fact that approaches as *B* goes to infinity amounts to saying that an empirical standard deviation approaches the population standard deviation as the number of replications grows large. The "population" in this case is the population of values , where

The ideal bootstrap estimate and its approximation are sometimes called *nonparametric bootstrap* estimates because they are based on *,* the non parametric estimate of the population *.* The parametric bootstrap uses a different estimate of *.*



**Fig.1 Schematic Diagram for the bootstrap algorithm for estimating standard errors**

**The number of bootstrap replications:**

Here are two rules of thumb, gathered from the authors' experience:

(1) Even a small number of bootstrap replications, say *B* = 25, is usually informative. *B* = 50 is often enough to give a good estimate of *.*

(2) Very seldom are more than *B* = 200 replications needed for estimating a standard error. However much bigger values of *B* are required for constructing bootstrap confidence intervals.

**The bootstrap estimate of bias:**

We assume the same setup as in the case of estimating the standard error. The *bias* of as an estimate of is defined to be the difference between the expectation of and the value of the parameter *,*

A large bias is usually an undesirable aspect of an estimator's performance. We are resigned to the fact that is a variable estimator of *,* but usually we don't want the variability to be overwhelmingly on the low side or on the high side. Unbiased estimates*,* those for which *,* play an important role in statistical theory and practice. They promote a nice feeling of scientific objectivity in the estimation process. Plug-in estimates aren't necessarily unbiased, but they tend to have small biases compared to the magnitude of their standard errors. This is one of the good features of the plug-in principle.

We can use the bootstrap to assess the bias of any estimator . The bootstrap estimate of biasis defined to be the estimate we obtain by substituting for in

Here , the plug-in estimate of may differ from . In other words, is the plug-in estimate of , whether or not is the plug-in estimate of *.*

For most statistics that arise in practice, the ideal bootstrap estimate must be approximated by Monte Carlo simulation. We generate independent bootstrap samples as in Figure 1, evaluate the bootstrap replications and

approximate the bootstrap expectation by the average

The bootstrap estimate of bias based on the *B* replications is,

Notice that the algorithm of Figure 1 applies exactly to calculation of except that at the last step we calculate rather than Of course we can calculate both and from the same set of bootstrap replications, and that is the procedure we follow in this paper. converges to *,* the ideal bootstrap estimate of bias, as *B* goes to infinity.

**Bias Corrected estimate:**

The primary reason to estimate bias is to correct by reducing its bias. If is an estimate of *,* then the bias-corrected estimatoris

-

Taking equal to gives

**Confidence intervals constructed based on bootstrapping:**

The technique of bootstrapping allows us to obtain estimates of bias and standard error of any practically conceivable statistic. It basically builds up an approximation of the true sampling distribution of the statistic using the observed distribution or histogram of the values of the statistic computed for each of the *B* bootstrap replicates, given by , known as the bootstrap distribution. The estimate of standard error is nothing but the standard deviation of the bootstrap distribution and the estimate of bias is nothing but a measure of the extent to which the mean of the bootstrap distribution is deviated from the plug-in estimate . Using the estimate of standard error as well as other quantities associated with the bootstrap distribution, such as its quantiles, various types of bootstrap confidence intervals are constructed. In this paper, we consider the standard(normal) confidence interval, Student’s t interval , Basic confidence interval , Bootstrap-t(Studentized) interval and the Percentile confidence interval.

**Standard (normal) confidence interval:**

Suppose that we are in the one-sample situation where the data are obtained by random sampling from an unknown distribution *,* . Let be the plug-in estimate of a parameter of interest and let be some reasonable estimate of standard error for , based perhaps on bootstrap computations. Under most circumstances it turns out that as the sample size *n* grows large, the distribution ofbecomes more and more normal, with mean near and variance near , written or equivalently where means ”asymptotically distributed as”

This large-sample, or asymptotic*,* usually holds true for general probability models as the amount of data gets large, and for statistics other than the plug-in estimate.

Let indicate the percentile point of a distribution.

If we take the normal approximation to be exact, then

Thus. in general

is called the standard confidence interval with coverage probability equal , or confidence level . Or, more simply, it is called a . confidence interval for .

Since we can write the interval in the more familiar form .

The coverage property of this interval implies that . of the time, a random interval constructed in this way will contain the true value *.* Of course this pivotality and normality of is only an approximation in most problems, and the standard interval is only an approximate confidence interval, though a very useful one in an enormous variety of situations. We will use the bootstrap to calculate better approximate confidence intervals.

**Student’s t interval:**

We proceed to discuss how the standard confidence interval can be improved upon. As we have seen, this interval is derived from the assumption that

.

This is valid as but is only an approximation for finite samples. Back in 1908, for the case *,* Gosset (Student) derived the better approximation

where represents the Student's *t* distribution with degrees of freedom. Using this approximation, our . confidence interval for is

with denoting the -th percentile of the *t* distribution with degrees of freedom. This method also assumes pivotality of and assumes it follows a *t* distribution with degrees of freedom.

The use of the *t* distribution doesn't adjust the confidence interval to account for skewness in the underlying population or other errors that can result when is not the sample mean.

**Basic Confidence Interval:**

To obtain the Basic confidence interval we consider the distribution of .If the distribution of W was known and is independent of ., then a two-sided confidence interval with confidence level could be obtained considering

which leads to the confidence interval

where -th percentile of the distribution of .

If we approximate the distribution of by , then the quantiles of the distribution of are approximated by the corresponding quantiles of .This leads to the Basic confidence interval

with confidence level . If be the -th percentile of the distribution of then . So the Basic CI takes the form

**Bootstrap-t (Studentized) interval:**

Through the use of the bootstrap we can obtain accurate intervals without having to make normal theory assumptions like in case of the standard(normal) confidence interval.

The bootstrap-t procedure estimates the distribution of directly from the data; in essence, it builds a table of quantiles that is appropriate for the data set at hand.This table is then used to construct a confidence interval in exactly the same way that the normal and *t* tables are used in constructing the the standard(normal) confidence interval and the Student’s t interval ,The bootstrap table is built by generating *B* bootstrap samples, and then computing the bootstrap version of *Z* for each. The bootstrap table consists of the percentiles of these *B* values.

We generate *B* bootstrap samples *,* and for each we compute

where is the value of for the bootstrap sample and is the estimated standard error of the bootstrap sample *.* The -th percentile of is estimated by the value such that

*£(a)* such that

For example, if *B* = 1000, the estimate of the 5% point is the 50th largest value of the ’*s* and the estimate of the 95% point is the 950th largest value of the ’*s .*Finally, the "bootstrap-t" confidence interval for is

with confidence level .

If is not an integer, the following procedure can be used. Assuming , let , the largest integer .

Then we define the empirical and quantiles by the -th largest and -th largest values of *),* respectively. 1 The idea behind the bootstrap-t method is easier to describe than the percentile-based bootstrap intervals which are described next. In practice, however, the bootstrap-t can give somewhat erratic results, and can be heavily influenced by a few outlying data points. The percentile based methods are more reliable.

the coverage of the bootstrap-t interval tends to be closer to the desired level than the coverage of the standard interval or the interval based on the *t* table. It is interesting that like

the *t* approximation, the gain in accuracy is at the price of generality. The standard normal table applies to all samples, and all sample sizes; the *t* table applies all samples of a fixed size n*;* the bootstrap-t table applies only to the given sample*.* However with the availability of fast computers, it is not impractical to derive a "bootstrap table"' for each new problem that we encounter.

Notice also that the normal and *t* percentage points are symmetric about zero, and as a consequence the resulting intervals are symmetric about the point estimate *B.* In contrast, the

bootstrap-t percentiles can by asymmetric about 0, leading to intervals which are longer on the left or right. This asymmetry represents an important part of the improvement in coverage that it enjoys.

There are both computational and interpretive problems with the bootstrap-t confidence procedure. In the denominator of the statistic we require *,.* For example, where is the mean, we use the plug-in estimate ,

being a bootstrap sample.

The difficulty arises when *B* is a more complicated statistic, for which there is no simple standard error formula. Standard error formulae exist for very few statistics, and thus we would need to compute a bootstrap estimate of standard error for each bootstrap sample*.* This implies two nested levels of bootstrap sampling. Now for the estimation of standard error,

*B* = 25 might be sufficient, while *B* = 1000 is needed for the computation of percentiles. Hence the overall number of bootstrap samples needed is perhaps 25 ·1000 = 25,000, a formidable number if is costly to compute.

Without going into further details, we note that bootstrap-t intervals are not transformation-respecting in the sense that under a transformation of the parameter of interest, the corresponding confidence interval cannot be mapped be mapped back to the interval constructed for the untransformed parameter.

**Percentile Interval:**

Let be the usual plug-in estimate of a parameter and be its estimated standard error. Consider the standard normal confidence interval . The endpoints of this interval can be described in a way that is particularly convenient for bootstrap calculations. Let indicate a random variable drawn from the distribution . Then and are the -th and -th

percentile points of

The previous discussion suggests how we might use the percentiles of the bootstrap histogram to define confidence limits. This is exactly how the percentile intervalworks. A bootstrap data set is generated according to some probability model and bootstrap replications are computed. Let be the cumulative distribution function of *.* Then percentile intervalis defined by the and percentiles of

Expressions (13.3) and (13.4) refer to the ideal bootstrap situation in which the number of bootstrap replications is infinite. In practice we must use some finite number *B* of replications. To proceed, we generate *B* independent bootstrap data and

compute the bootstrap replications *.*

Let be the 100.- th empirical percentile of the values

*.* So if and , is the 100th ordered value of the replications. (If *B* · *a* is not an integer, we may use the earlier convention ).

The approximate percentile interval is

If the bootstrap distribution of is roughly normal, then the standard normal and percentile intervals will nearly agree.

The percentile method automatically makes the appropriate transformation so that the bootstrap distribution becomes approximately normal.

The following result formalizes the fact that the percentile method always "knows" the correct transformation:

Percentile interval lemma. Suppose the transformation perfectly normalizes the distribution of

for some standard deviation c. Then the percentile interval based on equals

[

The percentile interval is transformation-respecting.The percentile interval for any (monotone) parameter transformation is simply the percentile interval for mapped by

It is also range-preserving, i.e. if the parameter under study is necessarily bounded within range, the percentile interval produces a confidence interval for the parameter that respects those bounds.

**Parametric bootstrap:**

In parametric bootstrap, we make the assumption that the distribution of the random variable under study X is well modelled by a parametric family of distributions

={,}, the functional form of which is known exactly up to a constant parameter , being the parameter space. The general theory of parametric bootstrap is exactly the same as in the case of nonparametric bootstrap except that the estimate of is now given by a parametric estimate instead of the nonparametric empirical CDF estimate . This parametric estimate may be the maximum likelihood estimate based on **.** If be the pmf or pdf corresponding to , then the maximum likelihood estimate of is a value such that the likelihood function or equivalently its logarithm is maximised. Then and we perform the exact same procedures as in the case of nonparametric bootstrapping with replaced by .

**Errors committed in Bootstrapping:**

Here we note that two types of errors are commited when using the bootstrap technique.

1. Statistical Error as we use in place of .
2. Simulation error as we replace expectation by summation.

The solutions to dealing with these two errors are :

1. Choosing the estimates of bias and standard error in such a way that the statistical error , though unavoidable, is reduced.
2. Choosing B, the number of bootstrap replications, large enough to minimise the simulation error
3. **Theoretical procedures and results used to apply classical procedures:**

* **Nonparametric inference:**

1. **Estimator of location parameter:**

Let X be a random variable having the distribution function F. Let be a random sample of size n drawn from the distribution F.

Let W be the set of the Walsh averages computed on the basis of , i.e.

The Hodges-Lehmann estimator of the location parameter of the distribution F is given by

In case of a symmetric distribution F, the Hodges-Lehmann estimator reduces to simply estimates the population median.

In case of non-symmetric distributions, the Hodges-Lehmann estimator estimates the pseudo-median.

As an estimate of the median in cases where the distribution may not be symmetric we propose as an estimator the sample median

There is no simple analytical form for the standard errors of these estimators to the best of my knowledge, so we estimate the standard errors by bootstrap technique only.

1. **Confidence intervals for Location Parameter:**

Let the Wilcoxon Signed rank test statistic be .

Let and denote respectively the largest and smallest integers such that the following two conditions hold.

and

Then an approximate level confidence interval for the location parameter is given by

where is the k–th smallest value in the increasing ordered arrangement of ’s.

The confidence interval is obtained by inverting the acceptance region of Wilcoxon Signed rank test.

In case of symmetric distribution, it gives a confidence interval for the median of the distribution.

In case the distribution may not be symmetric, we adopt the following procedure which is based on the inversion of the acceptance region of Sign test. Let be a random variable which follows a Binomial distribution with size parameter n and success probability p = 0.5 i.e. .

Let and denote respectively the largest and smallest integers such that the following two conditions hold.

and

Then an approximate level confidence interval for the location parameter is given by

where is the k–th smallest value in the increasing ordered arrangement of ’s.

* **Parametric inference:**

1. **Normal Distribution:**

Let . Let be a random sample of size n drawn from the distribution. The UMVUE of is given by and the standard error of is , which is estimated by where .

A level confidence interval for based on is given by

where is the -th percentile point of the distribution with degrees of freedom.

Another estimator of is proposed to be the sample median

The asymptotic standard error of is estimated by .

An approximate level confidence interval for based on is given by

where is the -th percentile point of the distribution.

For the parametric bootstrap for calculating the MLE’s used for

and are and , respectively.

1. **Exponential Distribution:**

Let . Let be a random sample of size n drawn from the distribution. The UMVUE of is given by and the standard error of is , which is estimated by where .

A level confidence interval for based on is given by

where is the -th percentile point of the distribution with degrees of freedom.

Another estimator of is proposed to be the sample median

The asymptotic standard error of is estimated by .

An approximate level confidence interval for based on is given by

where is the -th percentile point of the distribution.

For the parametric bootstrap for calculating the MLE used for is

1. **Cauchy Distribution:**

Let . Let be a random sample of size n drawn from the distribution. An estimator of is given by the sample median

The asymptotic standard error of is estimated by where denotes the sample interquartile range.

An approximate level confidence interval for based on is given by

where is the -th percentile point of the distribution.

For the parametric bootstrap for calculating the MLE’s used for

and are and ,

1. **Poisson Distribution:**

Let . Let be a random sample of size n drawn from the distribution. The UMVUE of is given by and the standard error of is , which is estimated by where .

An approximate level confidence interval for based on is given by

where is the -th percentile point of the distribution.

An approximate variance stabilized level confidence interval for based on is given by

where is the -th percentile point of the distribution.

For the parametric bootstrap for calculating the MLE used for is

1. **Zero-Truncated Poisson Distribution:**

Let . Let be a random sample of size n drawn from the distribution. An estimator of is the MLE given by which is the solution of in the equation , computed using fixed point iteration method. The asymptotic variance of is estimated by

An approximate level confidence interval for based on is given by

where is the -th percentile point of the distribution.

For the parametric bootstrap for calculating the MLE used for is .

1. **Simulation Study:**

All simulations were performed using R.

1. **Normal Distribution:**

The steps involved in the simulation are described as follows:

1. We fix the parameter =0 and the random seed is initiated at 1234.
2. We fix the parameter 1,5,10 taking one value at a time
3. We fix the sample size n= , taking one value at a time
4. We fix the significance level α = 0.05
5. We fix the number of bootstrap replications B=1000.
6. We simulate n observations from , given by
7. For non-parametric bootstrap, we randomly draw n observations from with replacement and compute all the estimates based on that bootstrap sample. For parametric bootstrap the MLE is calculated based on  **,** we randomly draw n observations from with replacement from the estimated parametric model and compute all the estimates based on that bootstrap sample. For calculation of standard error of estimates corresponding to each bootstrap sample, we repeat all the steps in Step vii. till now times. We set at 10.
8. We repeat Step vii. B times. This completes a single iteration of the simulation.
9. Step viii. is repeated for R iterations. We set R=100.
10. All point estimates such as bias, standard errors and MSE’s are averaged over the R iterations, i.e. summed over and divided by R. MSE’s are computed as the mean sum of squares of the difference between the estimated and true value of the parameter. Length of a Confidence interval is calculated by the difference between the upper and lower confidence limits. For a particular type of CI the average length is computed, averaging over the R iterations. The coverage probability of a particular type of CI is estimated by the proportion of times the CI covers the true value of the parameter out of the R iterations.
11. **Exponential Distribution:**

The steps involved in the simulation are described as follows:

1. The random seed is initiated at 12345.
2. We fix the parameter 0.1,1,5,10 taking one value at a time
3. We fix the sample size n= , taking one value at a time
4. We fix the significance level α = 0.05
5. We fix the number of bootstrap replications B=1000.
6. We simulate n observations from , given by

1. For non-parametric bootstrap, we randomly draw n observations from with replacement and compute all the estimates based on that bootstrap sample. For parametric bootstrap the MLE is calculated based on  **,** we randomly draw n observations from with replacement from the estimated parametric model and compute all the estimates based on that bootstrap sample.
2. We repeat Step vii. B times. This completes a single iteration of the simulation.
3. Step viii. is repeated for R iterations. We set R=100.
4. All point estimates such as bias, standard errors and MSE’s are averaged over the R iterations, i.e. summed over and divided by R. MSE’s are computed as the mean sum of squares of the difference between the estimated and true value of the parameter. Length of a Confidence interval is calculated by the difference between the upper and lower confidence limits. For a particular type of CI the average length is computed, averaging over the R iterations. The coverage probability of a particular type of CI is estimated by the proportion of times the CI covers the true value of the parameter out of the R iterations.
5. **Cauchy Distribution:**

The steps involved in the simulation are described as follows:

1. We fix the parameter =0 and the random seed is initiated at 1234.
2. We fix the parameter 1,5,10 taking one value at a time
3. We fix the sample size n= , taking one value at a time
4. We fix the significance level α = 0.05
5. We fix the number of bootstrap replications B=1000.
6. We simulate n observations from. given by
7. For non-parametric bootstrap, we randomly draw n observations from with replacement and compute all the estimates based on that bootstrap sample. For parametric bootstrap the MLE is calculated based on  **,** we randomly draw n observations from with replacement from the estimated parametric model and compute all the estimates based on that bootstrap sample.
8. We repeat Step vii. B times. This completes a single iteration of the simulation.
9. Step viii. is repeated for R iterations. We set R=100.
10. All point estimates such as bias, standard errors and MSE’s are averaged over the R iterations, i.e. summed over and divided by R. MSE’s are computed as the mean sum of squares of the difference between the estimated and true value of the parameter. Length of a Confidence interval is calculated by the difference between the upper and lower confidence limits. For a particular type of CI the average length is computed, averaging over the R iterations. The coverage probability of a particular type of CI is estimated by the proportion of times the CI covers the true value of the parameter out of the R iterations.
11. **Poisson Distribution:**

The steps involved in the simulation are described as follows:

1. The random seed is initiated at 1234.
2. We fix the parameter 0.1,1,5,10,50,100 taking one value at a time
3. We fix the sample size n= , taking one value at a time
4. We fix the significance level α = 0.05
5. We fix the number of bootstrap replications B=1000.
6. We simulate n observations from , given by

1. For non-parametric bootstrap, we randomly draw n observations from with replacement and compute all the estimates based on that bootstrap sample. For parametric bootstrap the MLE is calculated based on  **,** we randomly draw n observations from with replacement from the estimated parametric model and compute all the estimates based on that bootstrap sample.
2. We repeat Step vii. B times. This completes a single iteration of the simulation.
3. Step viii. is repeated for R iterations. We set R=100.
4. All point estimates such as bias, standard errors and MSE’s are averaged over the R iterations, i.e. summed over and divided by R. MSE’s are computed as the mean sum of squares of the difference between the estimated and true value of the parameter. Length of a Confidence interval is calculated by the difference between the upper and lower confidence limits. For a particular type of CI the average length is computed, averaging over the R iterations. The coverage probability of a particular type of CI is estimated by the proportion of times the CI covers the true value of the parameter out of the R iterations.
5. **Zero Truncated Poisson Distribution:**

The steps involved in the simulation are described as follows:

1. The random seed is initiated at 1234.
2. We fix the parameter 0.1,1,5,10,50,100 taking one value at a time
3. We fix the sample size n= , taking one value at a time
4. We fix the significance level α = 0.05
5. We fix the number of bootstrap replications B=1000.
6. We simulate n observations from

given by

1. For non-parametric bootstrap, we randomly draw n observations from with replacement and compute all the estimates based on that bootstrap sample. For parametric bootstrap the MLE is calculated based on  **,** we randomly draw n observations from with replacement from the estimated parametric model and compute all the estimates based on that bootstrap sample.
2. We repeat Step vii. B times. This completes a single iteration of the simulation.
3. Step viii. is repeated for R iterations. We set R=100.
4. All point estimates such as bias, standard errors and MSE’s are averaged over the R iterations, i.e. summed over and divided by R. MSE’s are computed as the mean sum of squares of the difference between the estimated and true value of the parameter. Length of a Confidence interval is calculated by the difference between the upper and lower confidence limits. For a particular type of CI the average length is computed, averaging over the R iterations. The coverage probability of a particular type of CI is estimated by the proportion of times the CI covers the true value of the parameter out of the R iterations.
5. **Tables and Results:**
6. **Normal Distribution:**

* **Nonparametric approach-**

**Estimator used: Hodges – Lehmann**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Hodges Lehmann Estimate | Bootstrap estimate of Bias | Bias-corrected estimate | Boot Estimate of standard error | MSE of HL estimator | MSE of Bias-corrected estimate |
| 1 | 10 | 0.0181819 | -0.015500 | 0.0336815 | 0.328985 | 0.0932668 | 0.1043 |
| 1 | 50 | 0.0149889 | 0.000804 | 0.0141850 | 0.145411 | 0.0249455 | 0.0254 |
| 1 | 100 | 0.0099501 | 0.000348 | 0.0096019 | 0.102704 | 0.0137035 | 0.0137 |
| 5 | 10 | 0.0749444 | 0.004019 | 0.0709251 | 0.727054 | 0.6236365 | 0.6341 |
| 5 | 100 | 0.0497503 | 0.001741 | 0.0480093 | 0.513520 | 0.3425864 | 0.3437 |
| 10 | 10 | 0.1818193 | -0.154996 | 0.3368149 | 3.289845 | 9.3266770 | 10.4283 |
| 10 | 50 | 0.1498889 | 0.008039 | 0.1418502 | 1.454109 | 2.4945460 | 2.5363 |
| 10 | 100 | 0.0995007 | 0.003482 | 0.0960186 | 1.027041 | 1.3703460 | 1.3746 |

**Results:** Here we observe that the bias values are very small in all cases except when =10 and n=10, and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. In that case the estimate also seems to be deviated from the true value of the parameter. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in .

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.342041 | 0.93 | 1.21222 | 0.89 | 1.399123 | 0.92 |
| 1 | 50 | 0.5806706 | 0.89 | 0.5700001 | 0.89 | 0.5844281 | 0.9 |
| 1 | 100 | 0.4065341 | 0.94 | 0.4025925 | 0.92 | 0.4075743 | 0.93 |
| 5 | 10 | 2.903353 | 0.89 | 2.85 | 0.89 | 2.922141 | 0.9 |
| 5 | 50 | 2.903353 | 0.89 | 2.85 | 0.89 | 2.922141 | 0.9 |
| 5 | 100 | 2.03267 | 0.94 | 2.012963 | 0.92 | 2.037871 | 0.93 |
| 10 | 10 | 13.42041 | 0.93 | 12.1222 | 0.89 | 13.99123 | 0.92 |
| 10 | 50 | 5.806706 | 0.89 | 5.700001 | 0.89 | 5.844281 | 0.9 |
| 10 | 100 | 4.065341 | 0.94 | 4.025925 | 0.92 | 4.075743 | 0.93 |

**Table 2**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Basic Bootstrap | | Studentized Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.165424 | 0.85 | 1.794254 | 0.92 | 1.154756 | 0.9 |
| 1 | 50 | 0.5671101 | 0.89 | 0.6795643 | 0.96 | 0.5671101 | 0.89 |
| 1 | 100 | 0.4014507 | 0.91 | 0.471647 | 0.97 | 0.4014507 | 0.93 |
| 5 | 10 | 2.83555 | 0.89 | 3.397822 | 0.96 | 2.83555 | 0.89 |
| 5 | 50 | 2.83555 | 0.89 | 3.397822 | 0.96 | 2.83555 | 0.89 |
| 5 | 100 | 2.007254 | 0.91 | 2.358235 | 0.97 | 2.007254 | 0.93 |
| 10 | 10 | 11.65424 | 0.85 | 17.94254 | 0.92 | 11.54756 | 0.9 |
| 10 | 50 | 5.671101 | 0.89 | 6.795643 | 0.96 | 5.671101 | 0.89 |
| 10 | 100 | 4.014507 | 0.91 | 4.71647 | 0.97 | 4.014507 | 0.93 |

**Results:** The Studentized Bootstrap/Bootstrap-t interval performs best in general in terms of empirical coverage. The basic bootstrap interval gives undercoverage in some cases compared to the others. Average lengths of CIs are comparable in all cases.

* **Parametric approach-**

**Estimator used: Sample Mean**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Sample mean | Estimate of standard error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample mean | MSE of Bias-corrected estimate |
| 1 | 10 | 0.0526078 | 0.300231 | -.0004832 | 0.0530909 | 0.284229 | 0.1138 | 0.1136 |
| 1 | 50 | 0.0118515 | 0.139021 | -.0002476 | 0.0120991 | 0.137855 | 0.0199 | 0.0201 |
| 1 | 100 | 0.0156089 | 0.099066 | 0.0001194 | 0.0154895 | 0.098673 | 0.0108 | 0.0108 |
| 5 | 10 | 0.2630388 | 1.501158 | -.0024158 | 0.2654546 | 1.421145 | 2.8450 | 2.8401 |
| 5 | 50 | 0.0592574 | 0.695105 | -.0012381 | 0.0604955 | 0.689278 | 0.4975 | 0.5013 |
| 5 | 100 | 0.0780446 | 0.495328 | 0.0005971 | 0.0774475 | 0.493365 | 0.2712 | 0.2710 |
| 10 | 10 | 0.5260776 | 3.002315 | -.0048316 | 0.5309092 | 2.842291 | 11.3799 | 11.3602 |
| 10 | 50 | 0.1185148 | 1.390209 | -.0024762 | 0.1209910 | 1.378555 | 1.9898 | 2.0053 |
| 10 | 100 | 0.1560892 | 0.990657 | 0.0011942 | 0.1548950 | 0.986730 | 1.0847 | 1.0838 |

**Results:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in .

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.100717 | 0.86 | 1.114158 | 0.87 | 1.285942 | 0.93 |
| 1 | 50 | 0.4661513 | 0.93 | 0.5403838 | 0.96 | 0.5540622 | 0.96 |
| 1 | 100 | 0.3289755 | 0.9 | 0.3867909 | 0.94 | 0.3915771 | 0.95 |
| 5 | 10 | 5.503583 | 0.86 | 5.570788 | 0.87 | 6.429708 | 0.93 |
| 5 | 50 | 2.330757 | 0.93 | 2.701919 | 0.96 | 2.770311 | 0.96 |
| 5 | 100 | 1.644878 | 0.9 | 1.933955 | 0.94 | 1.957886 | 0.95 |
| 10 | 10 | 11.007170 | 0.86 | 11.141580 | 0.87 | 12.859420 | 0.93 |
| 10 | 50 | 4.661513 | 0.93 | 5.403838 | 0.96 | 5.540622 | 0.96 |
| 10 | 100 | 3.289755 | 0.9 | 3.867909 | 0.94 | 3.915771 | 0.95 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.107147 | 0.87 | 1.107147 | 0.86 |
| 1 | 50 | 0.5365492 | 0.96 | 0.5365492 | 0.96 |
| 1 | 100 | 0.3853915 | 0.95 | 0.3853915 | 0.94 |
| 5 | 10 | 5.535735 | 0.87 | 5.535735 | 0.86 |
| 5 | 50 | 2.682746 | 0.96 | 2.682746 | 0.96 |
| 5 | 100 | 1.926957 | 0.95 | 1.926957 | 0.94 |
| 10 | 10 | 11.071470 | 0.87 | 11.071470 | 0.86 |
| 10 | 50 | 5.365492 | 0.96 | 5.365492 | 0.96 |
| 10 | 100 | 3.853915 | 0.95 | 3.853915 | 0.94 |

**Results:** The student’s t bootstrap interval performs best in terms of empirical coverage and consistency of coverage. Average lengths of CI’s are similar for all the intervals except for student’s t interval which is slightly longer on an average than the others.

**Estimator used: Sample Median**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Sample median | Estimate of standard error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample median | MSE of Bias-corrected estimate |
| 1 | 10 | 0.069020 | 0.376284 | -.016634 | 0.0856542 | 0.335284 | 0.1330 | 0.2237 |
| 1 | 50 | 0.008516 | 0.174237 | 0.003697 | 0.0048191 | 0.169850 | 0.0288 | 0.0544 |
| 1 | 100 | 0.005166 | 0.124160 | 0.009739 | -.0045733 | 0.122343 | 0.0167 | 0.0353 |
| 5 | 10 | 0.345102 | 1.881422 | -.083170 | 0.4282712 | 1.676417 | 3.3254 | 5.5923 |
| 5 | 50 | 0.042581 | 0.871185 | 0.018486 | 0.0240954 | 0.849248 | 0.7202 | 1.3602 |
| 5 | 100 | 0.025828 | 0.620802 | 0.048694 | -.0228664 | 0.611718 | 0.4178 | 0.8813 |
| 10 | 10 | 0.690203 | 3.762844 | -.166339 | 0.8565424 | 3.352835 | 13.3017 | 22.3694 |
| 10 | 50 | 0.085162 | 1.742369 | 0.036971 | 0.0481909 | 1.698496 | 2.8810 | 5.4409 |
| 10 | 100 | 0.051655 | 1.241604 | 0.097388 | -.0457327 | 1.223435 | 1.6712 | 3.5251 |

**Results:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in .

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| sigma | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.215283 | 0.89 | 1.295275 | 0.92 | 1.494985 | 0.96 |
| 1 | 50 | 0.5741682 | 0.91 | 0.6640464 | 0.94 | 0.680855 | 0.94 |
| 1 | 100 | 0.4056874 | 0.92 | 0.4750673 | 0.92 | 0.4809459 | 0.92 |
| 5 | 10 | 6.076416 | 0.89 | 6.476377 | 0.92 | 7.474925 | 0.96 |
| 5 | 50 | 2.870841 | 0.91 | 3.320232 | 0.94 | 3.404275 | 0.94 |
| 5 | 100 | 2.028437 | 0.92 | 2.375337 | 0.92 | 2.40473 | 0.92 |
| 10 | 10 | 12.15283 | 0.89 | 12.95275 | 0.92 | 14.94985 | 0.96 |
| 10 | 50 | 5.741682 | 0.91 | 6.640464 | 0.94 | 6.80855 | 0.94 |
| 10 | 100 | 4.056874 | 0.92 | 4.750673 | 0.92 | 4.809459 | 0.92 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| sigma | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.290314 | 0.81 | 1.290314 | 0.92 |
| 1 | 50 | 0.6619956 | 0.84 | 0.6619956 | 0.98 |
| 1 | 100 | 0.4734756 | 0.8 | 0.4734756 | 0.97 |
| 5 | 10 | 6.451569 | 0.81 | 6.451569 | 0.92 |
| 5 | 50 | 3.309978 | 0.84 | 3.309978 | 0.98 |
| 5 | 100 | 2.367378 | 0.8 | 2.367378 | 0.97 |
| 10 | 10 | 12.90314 | 0.81 | 12.90314 | 0.92 |
| 10 | 50 | 6.619956 | 0.84 | 6.619956 | 0.98 |
| 10 | 100 | 4.734756 | 0.8 | 4.734756 | 0.97 |

**Results:** The bootstrap t interval performs best in terms of coverage probabilities. However it is also longer on average than all the other intervals. The standard bootstrap interval provides consistent coverage as well as slightly shorter lengthed CI’s. The basic bootstrap interval consistently undercovers the true value of the parameter .

1. **Exponential Distribution:**

* **Nonparametric approach-**

**Estimator used: Sample Median**

**True Value of Median: ln(2)\*λ**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| λ | n | Sample median | Bootstrap estimate of Bias | Bias-corrected estimate | Boot Estimate of standard error | MSE of sample median | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.0713511 | 0.005322 | 0.0660288 | 0.0326726 | 0.0008628 | 0.0010 |
| 0.1 | 50 | 0.0681446 | 0.001474 | 0.0666705 | 0.0141003 | 0.0002218 | 0.0003 |
| 0.1 | 100 | 0.0682509 | 0.000520 | 0.0677311 | 0.0102798 | 0.0000791 | 0.0001 |
| 1.0 | 10 | 0.7135106 | 0.053223 | 0.6602878 | 0.3267262 | 0.0862769 | 0.1027 |
| 1.0 | 50 | 0.6814458 | 0.014741 | 0.6667048 | 0.1410027 | 0.0221846 | 0.0255 |
| 1.0 | 100 | 0.6825091 | 0.005198 | 0.6773114 | 0.1027976 | 0.0079069 | 0.0094 |
| 5.0 | 10 | 3.5675530 | 0.266114 | 3.3014390 | 1.6336310 | 2.1569230 | 2.5663 |
| 5.0 | 50 | 3.4072290 | 0.073705 | 3.3335240 | 0.7050134 | 0.5546156 | 0.6370 |
| 5.0 | 100 | 3.4125450 | 0.025988 | 3.3865570 | 0.5139878 | 0.1976724 | 0.2348 |
| 10.0 | 10 | 7.1351060 | 0.532228 | 6.6028780 | 3.2672620 | 8.6276930 | 10.2650 |
| 10.0 | 50 | 6.8144580 | 0.147410 | 6.6670480 | 1.4100270 | 2.2184620 | 2.5479 |
| 10.0 | 100 | 6.8250910 | 0.051977 | 6.7731140 | 1.0279760 | 0.7906896 | 0.9391 |

**Results:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases.. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
|  | Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.126905 | 0.94 | 0.1280743 | 0.9 | 0.1478212 | 0.95 |
| 0.1 | 50 | 0.05835118 | 0.94 | 0.05527204 | 0.88 | 0.0566711 | 0.9 |
| 0.1 | 100 | 0.04230668 | 0.97 | 0.0402959 | 0.96 | 0.04079453 | 0.96 |
| 1 | 10 | 1.26905 | 0.94 | 1.280743 | 0.9 | 1.478212 | 0.95 |
| 1 | 50 | 0.5835118 | 0.94 | 0.5527204 | 0.88 | 0.566711 | 0.9 |
| 1 | 100 | 0.4230668 | 0.97 | 0.402959 | 0.96 | 0.4079453 | 0.96 |
| 5 | 10 | 6.345251 | 0.94 | 6.403715 | 0.9 | 7.39106 | 0.95 |
| 5 | 50 | 2.917559 | 0.94 | 2.763602 | 0.88 | 2.833555 | 0.9 |
| 5 | 100 | 12.6905 | 0.94 | 12.80743 | 0.9 | 14.78212 | 0.95 |
| 10 | 10 | 5.835118 | 0.94 | 5.527204 | 0.88 | 5.66711 | 0.9 |
| 10 | 50 | 5.835118 | 0.94 | 5.527204 | 0.88 | 5.66711 | 0.9 |
| 10 | 100 | 4.230668 | 0.97 | 4.02959 | 0.96 | 4.079453 | 0.96 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| lambda | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.1203875 | 0.76 | 0.1203875 | 0.93 |
| 0.1 | 50 | 0.05437359 | 0.83 | 0.05437359 | 0.93 |
| 0.1 | 100 | 0.03936429 | 0.93 | 0.03936429 | 0.94 |
| 1 | 10 | 1.203875 | 0.76 | 1.203875 | 0.93 |
| 1 | 50 | 0.5437359 | 0.83 | 0.5437359 | 0.93 |
| 1 | 100 | 0.3936429 | 0.93 | 0.3936429 | 0.94 |
| 5 | 10 | 6.019375 | 0.76 | 6.019375 | 0.93 |
| 5 | 50 | 2.71868 | 0.83 | 2.71868 | 0.93 |
| 5 | 100 | 12.03875 | 0.76 | 12.03875 | 0.93 |
| 10 | 10 | 5.437359 | 0.83 | 5.437359 | 0.93 |
| 10 | 50 | 5.437359 | 0.83 | 5.437359 | 0.93 |
| 10 | 100 | 3.936429 | 0.93 | 3.936429 | 0.94 |

**Results:** The Classical Wald-type interval performs best in this case in terms of empirical coverages but is slightly longer in length than all the bootstrap intervals. The Basic bootstrap interval consistently undercovers to a large extent the true value of the parameter.

* **Parametric approach-**

**Estimator used: Sample Mean**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| λ | n | Sample mean | Estimate of standard error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample mean | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.1059 | 0.03102 | 0.0001525 | 0.1057 | 0.03353 | 0.0012 | 0.0012 |
| 0.1 | 50 | 0.1013 | 0.01365 | -.0000503 | 0.1014 | 0.01431 | 0.0003 | 0.0003 |
| 0.1 | 100 | 0.0998 | 0.00991 | -.0000636 | 0.0999 | 0.00998 | 0.0001 | 0.0001 |
| 1 | 10 | 1.0586 | 0.31018 | 0.0015250 | 1.0570 | 0.33531 | 0.1217 | 0.1215 |
| 1 | 50 | 1.0135 | 0.13651 | -.0005030 | 1.0140 | 0.14312 | 0.0280 | 0.0281 |
| 1 | 100 | 1.0028 | 0.09974 | -.0005660 | 1.0033 | 0.10009 | 0.0079 | 0.0080 |
| 5 | 10 | 5.2928 | 1.55092 | 0.0076240 | 5.2851 | 1.67657 | 3.0419 | 3.0367 |
| 5 | 50 | 5.0621 | 0.68531 | -.0018240 | 5.0639 | 0.71482 | 0.6215 | 0.6221 |
| 5 | 100 | 5.0138 | 0.49871 | -.0028290 | 5.0167 | 0.50044 | 0.1978 | 0.1988 |
| 10 | 10 | 10.5855 | 3.10184 | 0.0152500 | 10.5703 | 3.35314 | 12.1674 | 12.1468 |
| 10 | 50 | 10.1347 | 1.36513 | -.0050300 | 10.1397 | 1.43119 | 2.8003 | 2.8123 |
| 10 | 100 | 10.0277 | 0.99742 | -.0056600 | 10.0333 | 1.00089 | 0.7914 | 0.7952 |

**Results:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.1587853 | 0.95 | 0.1314407 | 0.94 | 0.1517066 | 0.96 |
| 0.1 | 50 | 0.05832237 | 0.91 | 0.0561015 | 0.88 | 0.05752156 | 0.91 |
| 0.1 | 100 | 0.03985697 | 0.99 | 0.0391184 | 0.98 | 0.03960246 | 0.99 |
| 1 | 10 | 1.587853 | 0.95 | 1.314407 | 0.94 | 1.517066 | 0.96 |
| 1 | 50 | 0.5832237 | 0.91 | 0.561015 | 0.88 | 0.5752156 | 0.91 |
| 1 | 100 | 0.4004737 | 0.97 | 0.39234 | 0.96 | 0.3971949 | 0.96 |
| 5 | 10 | 7.939264 | 0.95 | 6.572035 | 0.94 | 7.585331 | 0.96 |
| 5 | 50 | 2.913106 | 0.93 | 2.802055 | 0.92 | 2.872982 | 0.94 |
| 5 | 100 | 2.002368 | 0.97 | 1.9617 | 0.96 | 1.985974 | 0.96 |
| 10 | 10 | 15.87853 | 0.95 | 13.14407 | 0.94 | 15.17066 | 0.96 |
| 10 | 50 | 5.832237 | 0.91 | 5.61015 | 0.88 | 5.752156 | 0.91 |
| 10 | 100 | 4.004737 | 0.97 | 3.9234 | 0.96 | 3.971949 | 0.96 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| lambda | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 0.1298692 | 0.89 | 0.1298692 | 0.96 |
| 1 | 50 | 0.05582292 | 0.87 | 0.05582292 | 0.89 |
| 1 | 100 | 0.03897949 | 0.97 | 0.03897949 | 0.99 |
| 5 | 10 | 1.298692 | 0.89 | 1.298692 | 0.96 |
| 5 | 50 | 0.5582292 | 0.87 | 0.5582292 | 0.89 |
| 5 | 100 | 0.3907721 | 0.96 | 0.3907721 | 0.96 |
| 10 | 10 | 6.493459 | 0.89 | 6.493459 | 0.96 |
| 10 | 50 | 2.795002 | 0.9 | 2.795002 | 0.93 |
| 10 | 100 | 1.953861 | 0.96 | 1.953861 | 0.96 |

**Results:** The percentile bootstrap interval performs best in this case in terms of coverage probabilities as well as length of CI’s.

**Estimator used: Sample Median**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| λ | n | Sample median | Estimate of standard error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample median | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.1118 | 0.03347 | 0.001878 | 0.1100 | 0.04744 | 0.0026 | 0.0057 |
| 0.1 | 50 | 0.1043 | 0.01433 | -.001412 | 0.1057 | 0.02051 | 0.0005 | 0.0011 |
| 0.1 | 100 | 0.1002 | 0.00998 | 0.000248 | 0.1000 | 0.01440 | 0.0001 | 0.0004 |
| 1 | 10 | 1.1183 | 0.33474 | 0.018775 | 1.0995 | 0.47441 | 0.2590 | 0.5695 |
| 1 | 50 | 1.0426 | 0.14333 | -.014125 | 1.0568 | 0.20506 | 0.0537 | 0.1142 |
| 1 | 100 | 1.0082 | 0.10028 | 0.001496 | 1.0067 | 0.14426 | 0.0187 | 0.0466 |
| 5 | 10 | 5.5915 | 1.67372 | 0.093875 | 5.4977 | 2.37204 | 6.4743 | 14.2380 |
| 5 | 50 | 5.1920 | 0.71589 | -.054234 | 5.2462 | 1.02277 | 1.3296 | 3.1070 |
| 5 | 100 | 5.0411 | 0.50138 | 0.007480 | 5.0336 | 0.72129 | 0.4665 | 1.1658 |
| 10 | 10 | 11.1831 | 3.34744 | 0.187750 | 10.9953 | 4.74408 | 25.8972 | 56.9520 |
| 10 | 50 | 10.4263 | 1.43326 | -.141250 | 10.5676 | 2.05061 | 5.3677 | 11.4171 |
| 10 | 100 | 10.0822 | 1.00277 | 0.014960 | 10.0672 | 1.44257 | 1.8659 | 4.6633 |

**Results:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.1440427 | 0.88 | 0.1716981 | 0.92 | 0.198171 | 0.98 |
| 0.1 | 50 | 0.06665815 | 0.88 | 0.07891577 | 0.93 | 0.08091331 | 0.93 |
| 0.1 | 100 | 0.0472642 | 0.98 | 0.0561528 | 0.99 | 0.05684764 | 0.99 |
| 1 | 10 | 1.440427 | 0.88 | 1.716981 | 0.92 | 1.98171 | 0.98 |
| 1 | 50 | 0.6665815 | 0.88 | 0.7891577 | 0.93 | 0.8091331 | 0.93 |
| 1 | 100 | 0.474251 | 0.97 | 0.5620556 | 0.98 | 0.5690106 | 0.98 |
| 5 | 10 | 7.202137 | 0.88 | 8.584904 | 0.92 | 9.908551 | 0.98 |
| 5 | 50 | 3.302489 | 0.88 | 3.928248 | 0.92 | 4.027681 | 0.92 |
| 5 | 100 | 2.371255 | 0.97 | 2.810278 | 0.98 | 2.845053 | 0.98 |
| 10 | 10 | 14.40427 | 0.88 | 17.16981 | 0.92 | 19.8171 | 0.98 |
| 10 | 50 | 6.665815 | 0.88 | 7.891577 | 0.93 | 8.091331 | 0.93 |
| 10 | 100 | 4.74251 | 0.97 | 5.620556 | 0.98 | 5.690106 | 0.98 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| lambda | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.1661186 | 0.72 | 0.1661186 | 1 |
| 0.1 | 50 | 0.07885929 | 0.77 | 0.07885929 | 0.99 |
| 0.1 | 100 | 0.05580637 | 0.79 | 0.05580637 | 1 |
| 1 | 10 | 1.661186 | 0.72 | 1.661186 | 1 |
| 1 | 50 | 0.7885929 | 0.77 | 0.7885929 | 0.99 |
| 1 | 100 | 0.5616226 | 0.8 | 0.5616226 | 0.99 |
| 5 | 10 | 8.305932 | 0.72 | 8.305932 | 1 |
| 5 | 50 | 3.910777 | 0.76 | 3.910777 | 0.99 |
| 5 | 100 | 2.808113 | 0.8 | 2.808113 | 0.99 |
| 10 | 10 | 16.61186 | 0.72 | 16.61186 | 1 |
| 10 | 50 | 7.885929 | 0.77 | 7.885929 | 0.99 |
| 10 | 100 | 5.616226 | 0.8 | 5.616226 | 0.99 |

**Results:** The Percentile bootstrap interval performs best in this case in terms of both coverage probabilities and average lengths of CI’s. The basic bootstrap interval undercovers to a large extent the true value of the parameter.

1. **Cauchy Distribution:**

* **Nonparametric approach-**

**Estimator used: Hodges – Lehmann**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | n | Hodges Lehmann Estimate | Bootstrap estimate of Bias | Bias-corrected estimate | Boot Estimate of standard error | MSE of HL estimator | MSE of Bias-corrected estimate |
| 1 | 10 | -0.101473 | -0.12765 | 0.0261799 | 5.0278 | 0.4781 | 4.550 |
| 1 | 50 | -0.038171 | -0.00836 | -.0298069 | 0.2973 | 0.0747 | 0.068 |
| 1 | 100 | -0.029865 | -0.00112 | -.0287453 | 0.1966 | 0.0366 | 0.035 |
| 5 | 10 | -0.507366 | -0.63827 | 0.1308996 | 25.1388 | 11.9533 | 113.746 |
| 5 | 50 | -0.190853 | -0.04182 | -.1490343 | 1.4864 | 1.8682 | 1.699 |
| 5 | 100 | -0.149324 | -0.00560 | -.1437267 | 0.9831 | 0.9155 | 0.869 |
| 10 | 10 | -1.014733 | -1.27653 | 0.2617991 | 50.2775 | 47.8133 | 454.984 |
| 10 | 50 | -0.381706 | -0.08364 | -.2980686 | 2.9728 | 7.4729 | 6.795 |
| 10 | 100 | -0.298648 | -0.01119 | -.2874533 | 1.9662 | 3.6621 | 3.475 |

**Results:** A noticeable fact is that the Hodges-Lehmann estimator is consistently underestimating the parameter value**.** Here we observe that the bias values are very small in all cases .The MSE’s of the uncorrected estimates are much smaller than those of the corresponding bias corrected estimates by a factor of about 10. Thus here bias correction does not give is not at all advisable. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases.. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in .

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| sigma | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 41.45133 | 0.94 | 19.70842 | 0.99 | 22.74713 | 1 |
| 1 | 50 | 1.186895 | 0.94 | 1.16531 | 0.96 | 1.194807 | 0.96 |
| 1 | 100 | 0.7796267 | 0.92 | 0.7707202 | 0.92 | 0.7802573 | 0.93 |
| 5 | 10 | 207.25667 | 0.94 | 98.54211 | 0.99 | 113.73564 | 1 |
| 5 | 50 | 5.934473 | 0.94 | 5.82655 | 0.96 | 5.974033 | 0.96 |
| 5 | 100 | 3.898134 | 0.92 | 3.853601 | 0.92 | 3.901286 | 0.93 |
| 10 | 10 | 414.5133 | 0.94 | 197.0842 | 0.99 | 227.4713 | 1 |
| 10 | 50 | 11.86895 | 0.94 | 11.6531 | 0.96 | 11.94807 | 0.96 |
| 10 | 100 | 7.796267 | 0.92 | 7.707202 | 0.92 | 7.802573 | 0.93 |

**Table 2**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| sigma | n | Basic Bootstrap | | Studentized Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 4.60929 | 0.95 | 4.60929 | 0.91 | 4.60929 | 0.95 |
| 1 | 50 | 1.170455 | 0.97 | 1.170455 | 0.93 | 1.170455 | 0.97 |
| 1 | 100 | 0.7715222 | 0.97 | 0.7715222 | 0.91 | 0.7715222 | 0.97 |
| 5 | 10 | 23.04645 | 0.95 | 23.04645 | 0.91 | 23.04645 | 0.95 |
| 5 | 50 | 5.852277 | 0.97 | 5.852277 | 0.93 | 5.852277 | 0.97 |
| 5 | 100 | 3.857611 | 0.97 | 3.857611 | 0.91 | 3.857611 | 0.97 |
| 10 | 10 | 46.0929 | 0.95 | 46.0929 | 0.91 | 46.0929 | 0.95 |
| 10 | 50 | 11.70455 | 0.97 | 11.70455 | 0.93 | 11.70455 | 0.97 |
| 10 | 100 | 7.715222 | 0.97 | 7.715222 | 0.91 | 7.715222 | 0.97 |

**Results:** In this case the Basic and Percentile bootstrap intervals perform the best overall considering both coverage probabilities and average lengths of intervals.

* **Parametric approach-**

**Estimator used: Sample median**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| sigma | n | Sample median | Estimate of standard error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample median | MSE of Bias-corrected estimate |
| 1 | 10 | -.0127491 | 0.570727 | 0.020405 | -.0331544 | 0.612498 | 0.4518 | 0.6285 |
| 1 | 50 | 0.0157266 | 0.236262 | 0.003265 | 0.0124620 | 0.235734 | 0.0536 | 0.0771 |
| 1 | 100 | -.0174557 | 0.162909 | 0.005504 | -.0229597 | 0.160656 | 0.0268 | 0.0408 |
| 5 | 10 | -.8220594 | 2.706371 | 0.188220 | -.0102790 | 2.820870 | 7.1230 | 9.9967 |
| 5 | 50 | -.0029001 | 1.125345 | 0.060192 | -.0630919 | 1.124427 | 1.3672 | 2.1632 |
| 5 | 100 | 0.0529331 | 0.797464 | 0.009149 | 0.0437839 | 0.791367 | 0.7956 | 1.2086 |
| 10 | 10 | -.6329146 | 5.306426 | -.161932 | -.4709829 | 6.012063 | 37.9084 | 59.1167 |
| 10 | 50 | -.1021606 | 2.281738 | 0.105920 | -.2080804 | 2.245003 | 5.6013 | 8.9187 |
| 10 | 100 | -.0183299 | 1.554494 | 0.002420 | -.0207495 | 1.551068 | 2.7782 | 4.5323 |

**Results**: Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in .

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| sigma | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 1.471605 | 0.8 | 1.953876 | 0.87 | 2.25513 | 0.93 |
| 1 | 50 | 0.7528501 | 0.89 | 0.8962604 | 0.98 | 0.9189468 | 0.99 |
| 1 | 100 | 0.5323812 | 0.89 | 0.6221776 | 0.95 | 0.6298766 | 0.96 |
| 5 | 10 | 7.926923 | 0.82 | 9.769156 | 0.9 | 11.275394 | 0.91 |
| 5 | 50 | 3.567716 | 0.88 | 4.317304 | 0.93 | 4.426585 | 0.93 |
| 5 | 100 | 2.619066 | 0.84 | 3.144049 | 0.9 | 3.182954 | 0.9 |
| 10 | 10 | 16.3578 | 0.85 | 21.40206 | 0.92 | 24.70189 | 0.97 |
| 10 | 50 | 7.097106 | 0.87 | 8.479573 | 0.92 | 8.694211 | 0.92 |
| 10 | 100 | 5.004366 | 0.89 | 6.019125 | 0.97 | 6.093606 | 0.97 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| sigma | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 1 | 10 | 2.024315 | 0.84 | 2.024315 | 0.91 |
| 1 | 50 | 0.9096908 | 0.92 | 0.9096908 | 0.99 |
| 1 | 100 | 0.6262372 | 0.87 | 0.6262372 | 0.98 |
| 5 | 10 | 10.079454 | 0.87 | 10.079454 | 0.9 |
| 5 | 50 | 4.358731 | 0.84 | 4.358731 | 0.94 |
| 5 | 100 | 3.143758 | 0.83 | 3.143758 | 0.94 |
| 10 | 10 | 21.96315 | 0.86 | 21.96315 | 0.92 |
| 10 | 50 | 8.545492 | 0.84 | 8.545492 | 0.95 |
| 10 | 100 | 5.97834 | 0.89 | 5.97834 | 0.97 |

**Results:** Here the Student’s t bootstrap interval and percentile bootstrap interval perform better than the others in terms of coverage probabilities but they are slightly longer in length on average than the classical wald-type interval.

1. **Poisson Distribution:**

* **Nonparametric approach-**

**Estimator used: Sample median**

**True Value of Median(approximate):** λ

**Results of point estimation:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| λ | n | Sample median | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample median | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.000 | 0.010215 | -0.0102 | 0.041024 | 0.0064 | 0.0060 |
| 0.1 | 50 | 0.000 | 0.000000 | 0.0000 | 0.000000 | 0.0064 | 0.0064 |
| 0.1 | 100 | 0.000 | 0.000000 | 0.0000 | 0.000000 | 0.0064 | 0.0064 |
| 1.0 | 10 | 0.770 | 0.043380 | 0.7266 | 0.434943 | 0.1746 | 0.2788 |
| 1.0 | 50 | 0.965 | -0.061540 | 1.0265 | 0.211926 | 0.0523 | 0.1054 |
| 1.0 | 100 | 0.990 | -0.015820 | 1.0058 | 0.074874 | 0.0388 | 0.0535 |
| 5.0 | 10 | 4.765 | 0.041785 | 4.7232 | 0.883197 | 0.8603 | 1.1489 |
| 5.0 | 50 | 4.805 | -0.002300 | 4.8073 | 0.455059 | 0.1847 | 0.3439 |
| 5.0 | 100 | 4.935 | -0.056110 | 4.9911 | 0.346374 | 0.0865 | 0.1915 |
| 10.0 | 10 | 9.830 | 0.065135 | 9.7649 | 1.263563 | 1.6962 | 2.2476 |
| 10.0 | 50 | 9.915 | -0.013235 | 9.9282 | 0.623229 | 0.3643 | 0.5433 |
| 10.0 | 100 | 9.865 | -0.040495 | 9.9055 | 0.459888 | 0.1713 | 0.3297 |
| 50.0 | 10 | 50.020 | -0.054260 | 50.0743 | 2.811661 | 6.8996 | 9.3035 |
| 50.0 | 50 | 49.985 | 0.042340 | 49.9427 | 1.348707 | 1.6445 | 1.9909 |
| 50.0 | 100 | 49.705 | 0.028390 | 49.6766 | 0.939807 | 0.5537 | 0.7486 |
| 100.0 | 10 | 99.905 | 0.075680 | 99.8293 | 3.940091 | 13.3257 | 18.6020 |
| 100.0 | 50 | 100.005 | 0.067780 | 99.9372 | 1.850329 | 3.2417 | 3.9956 |
| 100.0 | 100 | 99.820 | -0.054970 | 99.8750 | 1.293797 | 1.1626 | 1.4111 |

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.05 | 0 | 0.1608111 | 0.21 | 0.1856055 | 0.27 |
| 0.1 | 50 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.1 | 100 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 10 | 1.53 | 0.97 | 1.704946 | 0.87 | 1.96782 | 0.95 |
| 1 | 50 | 0.78 | 0.74 | 0.8307367 | 0.79 | 0.8517646 | 0.8 |
| 1 | 100 | 0.29 | 0.29 | 0.2935023 | 0.29 | 0.2971341 | 0.29 |
| 5 | 10 | 4.37 | 0.98 | 3.462069 | 0.88 | 3.995861 | 0.91 |
| 5 | 50 | 1.66 | 0.95 | 1.7838 | 0.95 | 1.828952 | 0.96 |
| 5 | 100 | 1.14 | 0.81 | 1.357761 | 0.97 | 1.374562 | 0.97 |
| 10 | 10 | 6.52 | 0.95 | 4.953075 | 0.88 | 5.716756 | 0.93 |
| 10 | 50 | 2.44 | 0.95 | 2.443015 | 0.92 | 2.504853 | 0.93 |
| 10 | 100 | 1.68 | 0.97 | 1.802729 | 0.93 | 1.825036 | 0.94 |
| 50 | 10 | 15.74 | 0.96 | 11.02151 | 0.94 | 12.72084 | 0.96 |
| 50 | 50 | 5.66 | 0.94 | 5.286833 | 0.94 | 5.420655 | 0.94 |
| 50 | 100 | 3.75 | 0.97 | 3.683976 | 0.99 | 3.729562 | 0.99 |
| 100 | 10 | 22.12 | 0.97 | 15.44487 | 0.94 | 17.82621 | 0.96 |
| 100 | 50 | 7.89 | 0.98 | 7.253157 | 0.96 | 7.436752 | 0.96 |
| 100 | 100 | 5.4 | 0.97 | 5.071591 | 0.97 | 5.134348 | 0.97 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| lambda | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.12 | 0.19 | 0.12 | 0 |
| 0.1 | 50 | 0 | 0 | 0 | 0 |
| 0.1 | 100 | 0 | 0 | 0 | 0 |
| 1 | 10 | 1.53525 | 0.74 | 1.53525 | 0.89 |
| 1 | 50 | 0.585625 | 0.06 | 0.585625 | 0.59 |
| 1 | 100 | 0.175125 | 0 | 0.175125 | 0.19 |
| 5 | 10 | 3.326 | 0.75 | 3.326 | 0.96 |
| 5 | 50 | 1.510125 | 0.61 | 1.510125 | 0.92 |
| 5 | 100 | 1.060625 | 0.4 | 1.060625 | 0.74 |
| 10 | 10 | 4.685875 | 0.74 | 4.685875 | 0.94 |
| 10 | 50 | 2.181 | 0.77 | 2.181 | 0.88 |
| 10 | 100 | 1.565125 | 0.63 | 1.565125 | 0.92 |
| 50 | 10 | 10.75537 | 0.81 | 10.75537 | 0.95 |
| 50 | 50 | 5.0355 | 0.84 | 5.0355 | 0.9 |
| 50 | 100 | 3.465875 | 0.92 | 3.465875 | 0.97 |
| 100 | 10 | 14.88062 | 0.82 | 14.88062 | 0.93 |
| 100 | 50 | 7.00675 | 0.89 | 7.00675 | 0.95 |
| 100 | 100 | 4.851375 | 0.91 | 4.851375 | 0.96 |

**Results for point estimation:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results for interval estimation:** Due to the discreteness of the sampling distribution of the estimator of interest, erratic results are obtained in this case. This is because non parametric inference assumes continuity of the underlying distribution wherever the inferential methods are applied. Here the bootstrap distributions obtained are not smooth enough to yield satisfactory results. For very small values of the parameter of interest and small sample size, none of the intervals perform well. Although performance improves as a whole, nothing definite can be said about which interval performs the best in this case.

* **Parametric approach-**

**Estimator used: Sample mean**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Sample mean | Estimate of Standard Error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of sample mean | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.0940 | 0.07561 | 0.0003340 | 0.0937 | 0.0755031 | 0.008400 | 0.00840 |
| 0.1 | 50 | 0.0978 | 0.04293 | -.0000758 | 0.0979 | 0.0430675 | 0.001916 | 0.00191 |
| 0.1 | 100 | 0.1000 | 0.03110 | -.0000300 | 0.1000 | 0.0311407 | 0.001098 | 0.00111 |
| 1.0 | 10 | 0.9610 | 0.30577 | -.0002200 | 0.9612 | 0.3050646 | 0.089300 | 0.09014 |
| 1.0 | 50 | 0.9834 | 0.13985 | -.0000622 | 0.9835 | 0.1404483 | 0.021780 | 0.02164 |
| 1.0 | 100 | 1.0063 | 0.10016 | 0.0000230 | 1.0063 | 0.1003212 | 0.011901 | 0.01198 |
| 5.0 | 10 | 4.9510 | 0.70184 | -.0005310 | 4.9515 | 0.7009344 | 0.490500 | 0.49503 |
| 5.0 | 50 | 4.9720 | 0.31519 | -.0000960 | 4.9721 | 0.3154648 | 0.095744 | 0.09543 |
| 5.0 | 100 | 5.0231 | 0.22406 | -.0005340 | 5.0236 | 0.2239473 | 0.056285 | 0.05674 |
| 10.0 | 10 | 10.0750 | 1.00241 | 0.0017500 | 10.0733 | 0.9993524 | 1.076100 | 1.08600 |
| 10.0 | 50 | 9.8572 | 0.44390 | 0.0015760 | 9.8556 | 0.4440868 | 0.205808 | 0.20694 |
| 10.0 | 100 | 9.9784 | 0.31584 | 0.0008590 | 9.9775 | 0.3171406 | 0.107382 | 0.10694 |
| 50.0 | 10 | 50.4420 | 2.24542 | 0.0024700 | 50.4395 | 2.2445720 | 4.852400 | 4.81524 |
| 50.0 | 50 | 50.0346 | 1.00030 | -.0012700 | 50.0359 | 0.9994856 | 0.934188 | 0.93738 |
| 50.0 | 100 | 50.0166 | 0.70721 | 0.0001300 | 50.0165 | 0.7095645 | 0.420464 | 0.42004 |
| 100.0 | 10 | 99.5060 | 3.15409 | 0.0077600 | 99.4982 | 3.1575590 | 9.500000 | 9.48574 |
| 100.0 | 50 | 99.9610 | 1.41390 | 0.0011700 | 99.9598 | 1.4153890 | 2.062580 | 2.07327 |
| 100.0 | 100 | 99.9973 | 0.99997 | 0.0017300 | 99.9956 | 1.0020220 | 0.975113 | 0.97943 |

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Variance Stabilised Classical | | Standard Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.2963681 | 0.64 | 0.2963681 | 0.63 | 0.2959665 | 0.64 |
| 0.1 | 50 | 0.1682664 | 0.86 | 0.1682664 | 0.93 | 0.1688214 | 0.86 |
| 0.1 | 100 | 0.1218985 | 0.91 | 0.1218985 | 0.94 | 0.1220691 | 0.91 |
| 1 | 10 | 1.198604 | 0.92 | 1.198604 | 0.95 | 1.195831 | 0.92 |
| 1 | 50 | 0.5482037 | 0.95 | 0.5482037 | 0.94 | 0.5505473 | 0.95 |
| 1 | 100 | 0.3926338 | 0.89 | 0.3926338 | 0.89 | 0.3932519 | 0.91 |
| 5 | 10 | 2.751173 | 0.92 | 2.751173 | 0.92 | 2.747612 | 0.91 |
| 5 | 50 | 1.235521 | 0.95 | 1.235521 | 0.95 | 1.236599 | 0.95 |
| 5 | 100 | 0.8783004 | 0.88 | 0.8783004 | 0.88 | 0.8778574 | 0.88 |
| 10 | 10 | 3.929369 | 0.94 | 3.929369 | 0.95 | 3.91739 | 0.94 |
| 10 | 50 | 1.740067 | 0.93 | 1.740067 | 0.94 | 1.740788 | 0.93 |
| 10 | 100 | 1.238085 | 0.92 | 1.238085 | 0.94 | 1.243168 | 0.92 |
| 50 | 10 | 8.801869 | 0.96 | 8.801869 | 0.96 | 8.798561 | 0.96 |
| 50 | 50 | 3.921101 | 0.96 | 3.921101 | 0.96 | 3.917912 | 0.96 |
| 50 | 100 | 2.772209 | 0.96 | 2.772209 | 0.97 | 2.781442 | 0.97 |
| 100 | 10 | 12.3638 | 0.94 | 12.3638 | 0.94 | 12.37741 | 0.94 |
| 100 | 50 | 5.542391 | 0.94 | 5.542391 | 0.94 | 5.548224 | 0.94 |
| 100 | 100 | 3.919827 | 0.96 | 3.919827 | 0.96 | 3.927853 | 0.96 |

**Table 2**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Student’s t Bootstrap | | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.3415996 | 0.64 | 0.255025 | 0.64 | 0.255025 | 0.64 |
| 0.1 | 50 | 0.1730947 | 0.86 | 0.164035 | 0.86 | 0.164035 | 0.86 |
| 0.1 | 100 | 0.1235796 | 0.91 | 0.1201275 | 0.85 | 0.1201275 | 0.9 |
| 1 | 10 | 1.380208 | 0.94 | 1.188225 | 0.85 | 1.188225 | 0.92 |
| 1 | 50 | 0.5644829 | 0.97 | 0.54593 | 0.91 | 0.54593 | 0.92 |
| 1 | 100 | 0.3981181 | 0.91 | 0.3907 | 0.91 | 0.3907 | 0.89 |
| 5 | 10 | 3.171248 | 0.95 | 2.7296 | 0.91 | 2.7296 | 0.91 |
| 5 | 50 | 1.267901 | 0.96 | 1.231465 | 0.94 | 1.231465 | 0.94 |
| 5 | 100 | 0.8887202 | 0.88 | 0.8711 | 0.9 | 0.8711 | 0.89 |
| 10 | 10 | 4.521385 | 0.96 | 3.88175 | 0.94 | 3.88175 | 0.94 |
| 10 | 50 | 1.784852 | 0.94 | 1.737205 | 0.93 | 1.737205 | 0.93 |
| 10 | 100 | 1.258552 | 0.94 | 1.238652 | 0.93 | 1.238652 | 0.92 |
| 50 | 10 | 10.15515 | 1 | 8.7649 | 0.98 | 8.7649 | 0.96 |
| 50 | 50 | 4.017083 | 0.96 | 3.886475 | 0.95 | 3.886475 | 0.95 |
| 50 | 100 | 2.81586 | 0.97 | 2.762555 | 0.96 | 2.762555 | 0.97 |
| 100 | 10 | 14.28579 | 0.98 | 12.32237 | 0.95 | 12.32237 | 0.94 |
| 100 | 50 | 5.688663 | 0.96 | 5.536455 | 0.95 | 5.536455 | 0.95 |
| 100 | 100 | 3.976457 | 0.96 | 3.91583 | 0.97 | 3.91583 | 0.96 |

**Results for point estimation:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results for interval estimation:** The variance stabilized classical CI performs well in general while the student’s t interval works best in the higher sample sizes.

1. **Zero Truncated Poisson Distribution:**

* **Parametric approach-**

**Estimator used: MLE**

**Results of point estimation:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | MLE | Estimate of Standard Error | Boot estimate of Bias | Bias-corrected estimate | Bootstrap estimate of se | MSE of MLE | MSE of Bias-corrected estimate |
| 0.1 | 10 | 0.093 | 0.0916998 | 0.0006083 | 0.092 | 0.09313 | 0.0150935 | 0.0163333 |
| 0.1 | 50 | 0.097 | 0.0572668 | 0.0008904 | 0.096 | 0.05686 | 0.0034056 | 0.0035249 |
| 0.1 | 100 | 0.099 | 0.0423710 | 0.0002583 | 0.098 | 0.04202 | 0.0018382 | 0.0018961 |
| 1.0 | 10 | 0.966 | 0.3732358 | -.0154523 | 0.981 | 0.37089 | 0.1292301 | 0.1321697 |
| 1.0 | 50 | 0.971 | 0.1710736 | -.0027048 | 0.974 | 0.17147 | 0.0318875 | 0.0317793 |
| 1.0 | 100 | 1.001 | 0.1227001 | -.0017520 | 1.002 | 0.12274 | 0.0154196 | 0.0156049 |
| 5.0 | 10 | 4.941 | 0.7123803 | -.0064790 | 4.947 | 0.71369 | 0.5059724 | 0.5071648 |
| 5.0 | 50 | 4.971 | 0.3197434 | -.0014360 | 4.972 | 0.32019 | 0.0988991 | 0.0985336 |
| 5.0 | 100 | 5.023 | 0.2271655 | -.0010080 | 5.024 | 0.22706 | 0.0585769 | 0.0590668 |
| 10.0 | 10 | 9.929 | 0.9955639 | -.0007260 | 9.930 | 0.99445 | 0.9347276 | 0.9430501 |
| 10.0 | 50 | 9.955 | 0.4462014 | -.0003330 | 9.955 | 0.44691 | 0.1945589 | 0.1933096 |
| 10.0 | 100 | 10.029 | 0.3167130 | -.0007000 | 10.030 | 0.31637 | 0.1073506 | 0.1082940 |
| 50.0 | 10 | 49.846 | 2.2320860 | -.0013700 | 49.847 | 2.22818 | 4.7688000 | 4.8117860 |
| 50.0 | 50 | 49.917 | 0.9991205 | -.0011300 | 49.918 | 1.00027 | 0.9499200 | 0.9443937 |
| 50.0 | 100 | 50.080 | 0.7076524 | -.0017000 | 50.082 | 0.70660 | 0.5481730 | 0.5534977 |
| 100.0 | 10 | 99.763 | 3.1581470 | -.0031700 | 99.766 | 3.15287 | 9.6167000 | 9.7029870 |
| 100.0 | 50 | 99.884 | 1.4133630 | -.0018000 | 99.886 | 1.41478 | 1.9021440 | 1.8908970 |
| 100.0 | 100 | 100.113 | 1.0005520 | -.0024000 | 100.116 | 0.99936 | 1.0885650 | 1.0988450 |

**Results of interval estimation:**

**Table 1**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| lambda | n | Classical Wald-Type | | Standard Bootstrap | | Student’s t Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.3594568 | 0.38 | 0.3650755 | 0.38 | 0.4213639 | 0.38 |
| 0.1 | 50 | 0.2244818 | 0.92 | 0.2228745 | 0.92 | 0.228516 | 0.92 |
| 0.1 | 100 | 0.1660912 | 0.87 | 0.1647204 | 0.87 | 0.1667587 | 0.87 |
| 1 | 10 | 1.463057 | 0.9 | 1.453872 | 0.9 | 1.678035 | 0.91 |
| 1 | 50 | 0.6705963 | 0.96 | 0.6721478 | 0.96 | 0.6891614 | 0.96 |
| 1 | 100 | 0.4809757 | 0.95 | 0.4811509 | 0.94 | 0.4871048 | 0.94 |
| 5 | 10 | 2.792479 | 0.92 | 2.797599 | 0.93 | 3.228941 | 0.93 |
| 5 | 50 | 1.253371 | 0.96 | 1.255135 | 0.94 | 1.286905 | 0.96 |
| 5 | 100 | 0.8904724 | 0.89 | 0.8900578 | 0.89 | 0.9010716 | 0.9 |
| 10 | 10 | 3.902539 | 0.93 | 3.898156 | 0.93 | 4.499185 | 0.95 |
| 10 | 50 | 1.749077 | 0.93 | 1.751841 | 0.93 | 1.796184 | 0.95 |
| 10 | 100 | 1.241492 | 0.9 | 1.240161 | 0.9 | 1.255507 | 0.92 |
| 50 | 10 | 8.749616 | 0.95 | 8.734289 | 0.95 | 10.080968 | 0.96 |
| 50 | 50 | 3.91648 | 0.94 | 3.921 | 0.94 | 4.020249 | 0.95 |
| 50 | 100 | 2.773946 | 0.89 | 2.769833 | 0.89 | 2.804108 | 0.89 |
| 100 | 10 | 12.37971 | 0.94 | 12.35903 | 0.94 | 14.26458 | 0.96 |
| 100 | 50 | 5.540279 | 0.94 | 5.545842 | 0.93 | 5.686221 | 0.96 |
| 100 | 100 | 3.922091 | 0.89 | 3.917425 | 0.89 | 3.965899 | 0.89 |

**Table 2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| lambda | n | Basic Bootstrap | | Percentile Bootstrap | |
| Average Length | Coverage Probability | Average Length | Coverage Probability |
| 0.1 | 10 | 0.2732301 | 0.38 | 0.2732301 | 0.53 |
| 0.1 | 50 | 0.2068841 | 0.7 | 0.2068841 | 0.92 |
| 0.1 | 100 | 0.1590089 | 0.87 | 0.1590089 | 0.87 |
| 1 | 10 | 1.4237 | 0.9 | 1.4237 | 0.9 |
| 1 | 50 | 0.6649608 | 0.93 | 0.6649608 | 0.96 |
| 1 | 100 | 0.4790238 | 0.95 | 0.4790238 | 0.94 |
| 5 | 10 | 2.780578 | 0.92 | 2.780578 | 0.92 |
| 5 | 50 | 1.249331 | 0.95 | 1.249331 | 0.95 |
| 5 | 100 | 0.8838525 | 0.9 | 0.8838525 | 0.88 |
| 10 | 10 | 3.868073 | 0.93 | 3.868073 | 0.93 |
| 10 | 50 | 1.74308 | 0.94 | 1.74308 | 0.93 |
| 10 | 100 | 1.233559 | 0.9 | 1.233559 | 0.9 |
| 50 | 10 | 8.672975 | 0.95 | 8.672975 | 0.94 |
| 50 | 50 | 3.913055 | 0.93 | 3.913055 | 0.93 |
| 50 | 100 | 2.751287 | 0.89 | 2.751287 | 0.89 |
| 100 | 10 | 12.28965 | 0.94 | 12.28965 | 0.94 |
| 100 | 50 | 5.535565 | 0.94 | 5.535565 | 0.93 |
| 100 | 100 | 3.89217 | 0.89 | 3.89217 | 0.88 |

**Results for point estimation:** Here we observe that the bias values are very small in all cases and also the MSE’s of the uncorrected estimates and their bias corrected estimates are close. Thus here bias correction does not give any significant improvement. This can also be seen from the relative magnitudes of bias to the estimated standard errors, which are negligible in all cases. We observe that the estimate of standard error calculated using analytical formula and estimated using bootstrap technique are very close to each other in all cases. Estimated standard errors decrease with increase in sample size as expected. MSE’s are increasing with increase in λ.

**Results for interval estimation:** The Student’s t interval performs best in general in terms of coverage probabilities but is slightly large in length than the other intervals.

1. **Conclusion**

We thus see that the bootstrap technique is immensely useful in conjunction with classical statistical methods and can be considered as a modern tool in the arsenal of a statistician to solve a variety of statistical problems. However it has its limitations which must be kept in mind while applying them.

1. **References:**
2. Efron, B.; Tibshirani, R. (1993). An Introduction to the Bootstrap. Boca Raton, FL: Chapman & Hall/CRC. ISBN 0-412-04231-2.
3. Efron, B. (1981). "Nonparametric estimates of standard error: The jackknife, the bootstrap and other methods". Biometrika. 68 (3): 589–599. doi:10.1093/biomet/68.3.589.
4. Davison, A.C. and Hinkley, D.V. (1997) Bootstrap Methods and Their Application. In: Cambridge Series in Statistical and Probabilistic Mathematics, No. 1, Cambridge University Press, Cambridge. <http://dx.doi.org/10.1017/cbo9780511802843>
5. www.wikipedia.org
6. **Acknowledgement**

I would like to convey my gratitude towards supervisor, Professor Debjit Sengupta for his constant support and guidance over the course of this dissertation project. I would also like to thank my family and friends for their unconditional help and support.